Thesis proposal: Tensor networks to solve the quantum many-body problem and quantum field theories

start date – ideally September 2022
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The objective of this thesis will be to make progress in the field tensor networks, a powerful compression method, to then solve some of the hardest problems of theoretical physics. In particular, one of the objectives will be to attack strongly coupled quantum field theories with a (very recently introduced) continuum limit of these networks. The thesis will be primarily theoretical, but the objective is to obtain tangible numerical output at the end. **The thesis is funded**, by a PSL jeune équipe starting grant. It will be physically located at *Inria Paris*, within the Inria team QUANTIC a joint venture between Inria, Mines, and ENS Paris. Interested candidates should contact us asap, and can ideally start with an internship (possibly funded as well) to get a feel for the subject before considering a 3 year commitment.

Context

The laws of physics are currently known to an exquisite precision, but the *quantum many-body problem* makes it impossible to use them to predict what will happen in a wide range of situations. The physics of high temperature superconductivity, the physics of strong interactions in nuclei, the quantum chemistry of catalysts are three examples of hard problems where the laws are known, but where the quantum many-body problem makes quantitative predictions difficult or even impossible to obtain.

The quantum many-body problem comes from the fact that in a quantum system with N particles (or elementary degrees of freedom), the number of parameters needed to describe the quantum state explodes exponentially in N. Even representing the solution of the Schrödinger equation is out of reach. For example, considering a $4 \times 4 \times 4$ arrangement of spins in a 3d quantum magnet, it is impossible to directly solve the Schrödinger equation (even using *Fugaku*, the most powerful supercomputer on Earth).

There are two modern approaches to attack this problem:

1. Build quantum computers, that encode the state directly in quantum bits, and thus do not need an exponentially large memory,

2. Use a classical compression approach to sparsely represent the quantum state with a much smaller number of parameters

For the second approach, the most recent and promising technique is *tensor networks*. Currently, they have been used to solve prototypical problems that were believed to be out of reach. This started a huge interest in the method, in theoretical physics and even in machine learning.

The objective of the thesis

Currently, tensor networks have been extremely powerful to solve “toy” problems on the lattice. The objective will be to extend this success to more realistic continuum models (relativistic quantum field theories, quantum gases), on the path towards the hardest problems (*e.g.* Quantum Chromodynamics).

To this end, one can take the continuum limit numerically, working with a tensor network on a very fine lattice, or work directly in the continuum, by taking the continuum limit of tensor networks theoretically first. The student will explore both options, and in particular the latter one, which has been pushed recently by us. It is a great occasion to learn a lot of non-trivial quantum field theory, through a fairly new and unusual lens.

Tensor networks can be used to solve the Schrödinger equation, but they have also been shown to allow some of the most numerically accurate implementations of the renormalization group. Depending on the student preferences, working on the extension of such tensor inspired renormalization group methods to continuum models is a natural additional line of inquiry of the thesis.

More generally, there will be a lot of freedom during the PhD to make the subject evolve to fit the tastes of the students or to adapt to parallel progress in this quickly evolving field. The student will also be encouraged to develop side interests in neighboring fields (Hamiltonian truncation method, conformal bootstrap, quantum information theory, quantum computing, etc.)

Ideal skills

The ideal PhD student will have a remarkable scholar track record, strong theoretical physics background (including, ideally, some quantum field theory and quantum information theory), be autonomous, and have a taste both for hardcore theory and numerics. Naturally, no candidate is expected to have the ideal set of skills! Thus, very motivated applicants, regardless of background, should contact us.

Environment

The PhD student will be physically located at *Inria Paris*, in the équipe QUANTIC, which is a joint venture between Inria, Mines ParisTech, and ENS Paris. The group recently developed a more fundamental theory effort, which is the focus of the present thesis proposal. However, its main activity is in the development of new theoretical and experimental techniques to control superconducting qubits better, on the way toward a universal quantum computer. Transverse explorations during the thesis, more related to these core interests of the group, are possible.

For the tensor network aspects, we have strong connections with ENS Paris, and groups abroad. In particular, the student will be encouraged to collaborate with strong groups at the University of Ghent (Belgium) and the Max Planck Institute of Quantum Optics (Munich, Germany).

Internship possibility

Starting a 3 year PhD is a strong and to some extent risky commitment, both for the student and us. We thus encourage interested applicants to contact us asap for a possible internship with us (that can be funded), that can be an excellent way to get a feel for the subjects and see how people in the group work.